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Also solved by H. C. FEEMSTER, J. A. CAPARO, CLIFFORD N. MILLS, and HORACE OLSON. Erroneously solved by HERBERT N. CARLETON.

306. Proposed by EMMA M. GIBSON, Drury College.

A sphere is composed of a solid homogeneous hemisphere and a very thin hemispherical shell of equal mass. What is the greatest inclination of a rough plane on which the sphere can just rest in equilibrium?

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

Let the radius of the sphere be a , the center at O , the center of gravity at G ; then

$$OG = \frac{1}{2} \left(\frac{a}{2} - \frac{3a}{8} \right) = \frac{a}{16};$$

however the sphere is placed, G is on a circumference of radius $a/16$, centered at O . If the sphere is in equilibrium, G is vertically over the contact, C , of the sphere with the inclined plane on which it rests; the inclination of the plane is the angle COG , which has its greatest value when CG is tangent to the little circle; this value is $\sin^{-1} 1/16$. Hence, the greatest inclination of the plane is $\sin^{-1} 1/16$ or $\tan^{-1} \mu$ (μ being the coefficient of friction) whichever happens to be the less.

Also solved by J. A. CAPARO. Erroneously solved by HERBERT N. CARLETON.

NUMBER THEORY.

226. Proposed by ELBERT H. CLARKE, Purdue University.

If $0!$ is taken equal to 1, and if k is any positive integer greater than or equal to 2, show that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \frac{1}{k-1} \cdot \frac{1}{(k-1)!}.$$

I. SOLUTION BY S. A. JOFFE, New York City.

Since

$$\frac{n!}{(k+n)!} = \frac{1 \cdot 2 \cdot 3 \cdots n}{1 \cdot 2 \cdot 3 \cdots n(n+1)(n+2) \cdots (n+k)} = \frac{1}{(n+1)(n+2) \cdots (n+k)},$$

we see that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2) \cdots (n+k)}. \quad (1)$$

Now, the series in the second member, each term of which is the reciprocal of a factorial expression, may be easily summed by an elementary theorem in finite differences or by the following equivalent simple method.

Noticing that

$$\begin{aligned} \frac{1}{(n+1)(n+2) \cdots (n+k-1)} - \frac{1}{(n+2)(n+3) \cdots (n+k)} \\ = \frac{(n+k) - (n+1)}{(n+1)(n+2) \cdots (n+k)} = \frac{k-1}{(n+1)(n+2) \cdots (n+k)}, \end{aligned}$$

we find that

$$\begin{aligned} \frac{1}{(n+1)(n+2) \cdots (n+k)} \\ = \frac{1}{k-1} \left[\frac{1}{(n+1)(n+2) \cdots (n+k-1)} - \frac{1}{(n+2)(n+3) \cdots (n+k)} \right]. \end{aligned}$$